Optically Compensated Zoom Lens

Glenn Wooters and E. W. Silvertooth

Reprinted from Journal of the Optical Society of America, Vol. 55, No. 4, pp. 347–351, April 1965

Reprinted from JOURNAL OF THE OPTICAL SOCIETY OF AMERICA, Vol. 55, No. 4, 347–351, April, 1965
Printed in U. S. A.

Optically Compensated Zoom Lens

GLENN WOOTERS AND E. W. SILVERTOOTH*
Nortronics, a Division of Northrop Corporation, 500 East Orangethorpe Avenue, Anaheim, California 92801
(Received 27 July 1964)

Investigation of the thin lens theory of zoom lenses results in a general statement concerning conjugate points, a simple proof of the maximum number of crossing points, and an algorithm for computing component focal lengths of a five-component symmetrical zoom lens. The three-component optically compensated zoom lens is discussed in detail. Results of applying the algorithm are given. A prototype of the five-component zoom lens has been built and is briefly discussed.

INTRODUCTION

THE thin lens treatment of the optically compensated zoom lens will be introduced by considering systems symmetrical about the midtravel position, that is, systems of magnification range M bounded at $M^{-\frac{1}{2}}$ and $M^{\frac{1}{2}}$. Further, only components participating in the zoom effect are considered, since components redundant to the effect transform fixed conjugates and may be separately treated by conventional means. Such exclusion leads additionally to a useful clarity of nomenclature and predictability of behavior.

* Present address: Custom Instruments, Inc., 974 Flintridge Avenue, Flintridge, California.

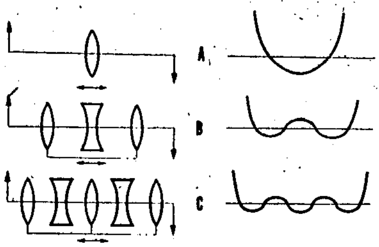

FIG. 1. Number of possible crossing points with one-, three- and five-component optically compensated zoom lenses.

The simplest zoom lens consists of a single component operating as shown in Fig. 1(A).

This system has two crossings; that is, the distance from object to image is the same at two magnifications. Also, there are two finite variable air spaces. Ignoring redundant components, we see that the next system in order of complexity [Fig. 1(B)] contains three components and four finite variable air spaces. In general, any zoom system as defined will have an odd number of components and even number of variable air spaces.

NUMBER OF CROSSING POINTS

It is of interest to determine next the maximum number of crossings possible, to extend consideration to the nonsymmetrical case including infinite conjugates, and to include the case of linear compensation. The matrix method for paraxial rays is useful for this purpose.[1]

The three-component zoom lens can be represented as in Fig. 2. The matrix method shows clearly that the number of possible crossing points is directly associated with the number of variable air spaces. The matrix for this system can be expressed as follows:

$$\begin{pmatrix}1 & -t_0\\0 & 1\end{pmatrix}\begin{pmatrix}1 & 0\\A_1 & 1\end{pmatrix}\begin{pmatrix}1 & -t_1\\0 & 1\end{pmatrix}\begin{pmatrix}1 & 0\\A_2 & 1\end{pmatrix}$$
$$\times\begin{pmatrix}1 & -t_2\\0 & 1\end{pmatrix}\begin{pmatrix}1 & 0\\A_3 & 1\end{pmatrix}\begin{pmatrix}1 & -t_3\\0 & 1\end{pmatrix}=\begin{pmatrix}B & D\\A & C\end{pmatrix},$$

where $A_1=1/f_1$, $A_2=1/f_2$, and $A_3=1/f_3$. When t_0 and t_3 are conjugate distances, then D in the matrix is equal to zero. Consider a movement of lenses f_1 and f_3 of an amount Δ. The spacings become $t_0+\Delta$, $t_1-\Delta$, $t_2+\Delta$, and $t_3-\Delta$. For how many values of Δ may $D=0$? The algebra is greatly simplified if only the maximum power

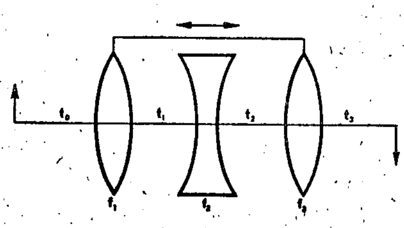

FIG. 2. Three-component zoom lens with negative fixed component.

[1] T. Smith, Proc. Phys. Soc. (London) 57, 558 (1945).

of Δ is retained in the matrix multiplication. Then the spacing matrix

$$\begin{pmatrix}1 & -t+\Delta\\0 & 1\end{pmatrix}$$

can be written as

$$\begin{pmatrix}1 & \Delta\\0 & 1\end{pmatrix}.$$

The power matrix

$$\begin{pmatrix}1 & 0\\A & 1\end{pmatrix}$$

can be written as

$$\begin{pmatrix}1 & 0\\1 & 1\end{pmatrix}$$

when the powers of Δ are retained.[2] In multiplication only the maximum power of Δ is to be retained, so

$$\begin{pmatrix}1 & \Delta\\0 & 1\end{pmatrix}\begin{pmatrix}1 & 0\\1 & 1\end{pmatrix}=\begin{pmatrix}1+\Delta & \Delta\\1 & 1\end{pmatrix}\rightarrow\begin{pmatrix}\Delta & \Delta\\1 & 1\end{pmatrix}.$$

The three-component zoom then is represented by

$$\begin{pmatrix}1 & \Delta\\0 & 1\end{pmatrix}\begin{pmatrix}1 & 0\\1 & 1\end{pmatrix}\begin{pmatrix}1 & \Delta\\0 & 1\end{pmatrix}\begin{pmatrix}1 & 0\\1 & 1\end{pmatrix}\begin{pmatrix}1 & \Delta\\0 & 1\end{pmatrix}$$
$$\times\begin{pmatrix}1 & 0\\1 & 1\end{pmatrix}\begin{pmatrix}1 & \Delta\\0 & 1\end{pmatrix}\rightarrow\begin{pmatrix}\Delta^3 & \Delta^4\\\Delta^2 & \Delta^3\end{pmatrix}.$$

The maximum power of Δ to appear in D is four, so there can be a maximum of four crossing points. Each factor Δ which appears in D arises from a variable air space. A five-component zoom lens will have two more variable air spaces and thus two more possible crossing points. The matrix method yields a similar result:

$$\begin{pmatrix}\Delta^3 & \Delta^4\\\Delta^2 & \Delta^3\end{pmatrix}\begin{pmatrix}1 & 0\\1 & 1\end{pmatrix}\begin{pmatrix}1 & \Delta\\0 & 1\end{pmatrix}\begin{pmatrix}1 & 0\\1 & 1\end{pmatrix}\begin{pmatrix}1 & \Delta\\0 & 1\end{pmatrix}\rightarrow$$
$$\begin{pmatrix}\Delta^3 & \Delta^4\\\Delta^2 & \Delta^3\end{pmatrix}\begin{pmatrix}\Delta & \Delta^2\\\Delta & \Delta^2\end{pmatrix}\rightarrow\begin{pmatrix}\Delta^5 & \Delta^6\\\Delta^4 & \Delta^5\end{pmatrix}.$$

A three-component zoom lens has only three possible crossing points when the object is at infinity. This comes about since a change in the position of the front component does not change the convergence or divergence of the light striking the component and thus the air space in front of the component is not a variable air space.

A slight extension of the above argument proves that linearly compensated zoom lenses may have the same number of crossing points as finite variable air

[2] The substitution of 1 for A is justified since we are interested in the highest powers of Δ resulting from multiplication. Multiplying some power of Δ by either 1 or A gives the same result as far as the highest power of Δ is involved.

spaces. The spaces in a three-component linearly compensated zoom lens can be represented as $t_0+a\Delta$, $t_1+b\Delta$, $t_2+c\Delta$, and $t_3+d\Delta$ when a, b, c, and d are constants such that $a+b+c+d=0$. In the matrix multiplication, the matrix for the first air space is

$$\begin{pmatrix} 1 & -t-a\Delta \\ 0 & 1 \end{pmatrix}.$$

Now if only the powers of Δ are retained, and all nonessential information is dropped, the matrix can be written

$$\begin{pmatrix} 1 & \Delta \\ 0 & 1 \end{pmatrix}.$$

This, however, is identical to the first term for the optically compensated lens and the remainder of the proof is identical.

LOCATION OF CONJUGATE POINTS

Another expression necessary for later development gives the object distance for a zoom system. Consider an optical system with a reference plane in the object space and a reference plane in the image space. The paraxial matrix is

$$\begin{pmatrix} B & D \\ A & C \end{pmatrix}.$$

If the light from the object is striking the first reference plane with a divergence of A_0, we can then write

$$\begin{pmatrix} 1 & 0 \\ A_0 & 1 \end{pmatrix}\begin{pmatrix} B & D \\ A & C \end{pmatrix}=\begin{pmatrix} B & D \\ A_0B+A & A_0D+C \end{pmatrix}.$$

The image will be formed at a distance $(A_0D+C)/(A_0B+C)$ from the reference plane in the image space.

Now consider the case when some component or components are shifted within the lens system. The new paraxial matrix can be represented as

$$\begin{pmatrix} B' & D' \\ A' & C' \end{pmatrix}.$$

The back conjugate for the same object location will be

$$(A_0D'+C')/(A_0B'+A').$$

For the system to work as a zoom lens in the two configurations, then

$$(A_0D'+C')/(A_0B'+A')=(A_0D+C)/(A_0B+A).$$

The equation is a quadratic in A_0. The system can have two different sets of conjugates and act as a zoom system in the two configurations. Nothing is implied about whether or not the same conjugates will apply for any other configuration of the lens system. Also, no limitation was placed on how the configuration of the lens system was changed. We can thus arrive at a general statement which applies to any type of zoom system.

Any lens system working in the zoom mode will function for two sets of conjugates at the most. More likely, the system will work for only one set of conjugates.

THREE-COMPONENT ZOOM

The three-component zoom is one which can be investigated thoroughly. Consider again the system in Fig. 2. We will arbitrarily set $f_2=-100$ and $t_1=t_2=60$ for the nominal position. The motion of the outside elements will be ± 50.

Since there may be four finite variable air spaces there can be four crossing points. One crossing point may lie outside the range of motion, or may disappear should the object distance or image distance become infinity. Systems with crossing points occurring at the end of travel and at the midposition will be investigated, although another crossing point may exist. The solutions were found by trying values for f_1 and f_3, finding the conjugates for the ends of the travel, and then finding the error at the midposition of travel. Either f_1 or f_3 was changed to reduce the error. Those solutions for no error in the midposition are plotted in Fig. 3. Rather than plot according to f_1 and f_3, the diopter powers of the lenses are used. A number of solutions are shown which are not mentioned in any of the literature we searched. In the first quadrant two curves are shown. These are the two solutions for the quadratic in A_0. The closed loop is for the case when one conjugate is real. The other curve is for the case where both object and image are virtual. The closed loop is for the ordinary object and image, while the other curve is for the pupils. In the present case it is possible to find solutions where not only are the object and image relatively fixed, but the pupils are also relatively fixed.

The solutions of most interest lie on the closed loop in the first quadrant between the two indicated points.

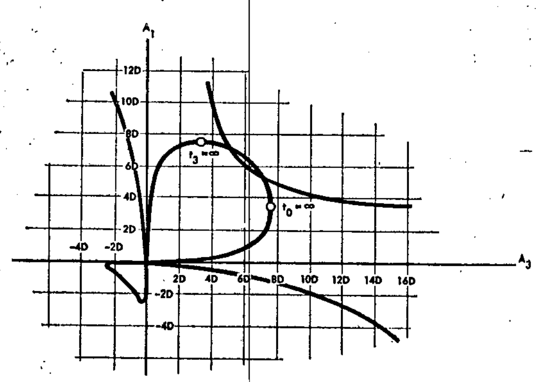

FIG. 3. Solutions for a three-component zoom lens with crossing points occurring at the midposition and at the two ends of travel (negative fixed component) $A_2=-10D$, $t_1=t_2=60$ mm, shift ± 50 mm.

Approved For Release 2002/01/02 : CIA-RDP78B04747A002300080016-2

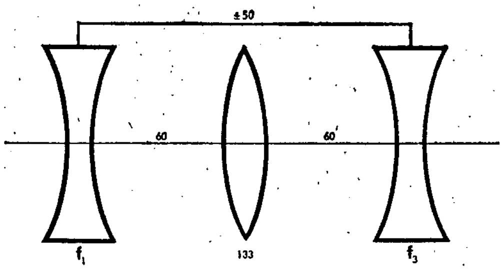

FIG. 4. Three-component zoom lens with positive fixed component.

At one of these points the object is at infinity while at the other the image is at infinity. For portions of the curve between the indicated points, the system has a real object and a real image. The symmetrical case ($f_1=f_3$) is of interest since it has the greatest magnification range, can have four useful crossing points, and has the smallest errors of focus between crossing points.

The solutions in the other quadrants are largely of academic interest. Those in the third quadrant consist of three negative components, and have small ranges of magnification.

The three-component zoom lens in which the fixed component is positive can be investigated in a similar fashion. The lens is shown in Fig. 4. In this case $f_2=133$, while $t_1=t_2=60$ for the nominal case. The motion is again ± 50.

The solutions are plotted in Fig. 5. The useful solutions are in quadrant 3. No solution corresponding to fixed pupils was found with the positive fixed component. Again the part of the curve of greatest interest lies between the two points where the curve is tangent to the two coordinate axes. At one point of tangency the object is at infinity while at the other point of tangency the image is at infinity.

The object and image are virtual for all values on the curve between the two tangent points. Again the symmetrical case ($f_1=f_3$) is capable of four crossing points,

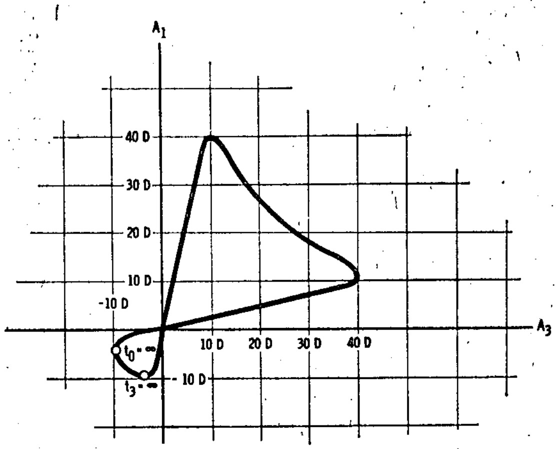

FIG. 5. Solutions for a three-component zoom lens with crossing points occurring at the midposition and at the two ends of travel (positive fixed component), $A_2=7.5\ D$, $t_1=t_2=60$ mm, shift ± 50 mm.

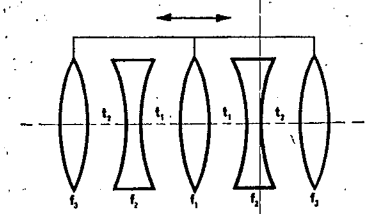

FIG. 6. Symmetrical five-component zoom lens with negative fixed components.

has the maximum magnification range, and has the smallest focusing errors between crossing points.

FIVE-COMPONENT ZOOM

The investigation of the three-component zoom showed the superiority of the symmetrical case. By analogy it is to be expected that the symmetrical case in the five-component zoom would be desirable. The exact conjugates and magnifications required can be obtained by using auxiliary components. If an unsymmetrical five-component zoom lens were desired, the best approach would probably be to find the symmetrical solution and modify it.

The notation used for the five-component, optically compensated zoom lens is shown in Fig. 6. In the illustrative examples $t_1=t_2=60$ for the nominal, or one-to-one case. The range of motion is ± 50. The system can give six crossing points. This is checked by finding the conjugates for ± 10, ± 30, and ± 50 and comparing. If the matrix for $+10$ motion is

$$\begin{pmatrix} B & D \\ A & C \end{pmatrix},$$

and if the reference planes are placed symmetrically, then the matrix for -10 motion is

$$\begin{pmatrix} C & D \\ A & B \end{pmatrix}.$$

The five-component, optically compensated zoom system can be determined by the following computational steps. Fix the values for f_1, t_1, and t_2. Assume values for f_2 and f_3. These will have to be of reasonable magnitude or the resulting solutions may be of little interest. Decide where the system is to have crossing points. Suppose these are at $\pm a$, $\pm b$, and $\pm c$. Call these position A, position B, and position C. Further

TABLE I. Five-component zoom—positive components movable.

f_1	f_2	f_3	Object distance	Mag. range	Max. error
140	−123.6	217.7	599	6.0	0.003
120	−101.4	185.8	511	8.4	0.007
100	−79.3	154.2	423	14	0.016
80	−57.2	123.0	336	30	0.046
60	−35.3	92.3	252	128	0.24
40	−13.3	62.8	173	5300	2.86

TABLE II. Five-component zoom—negative components movable.

f_1	f_2	f_3	Object distance	Mag. range	Max. error
−120	165.7	−202.6	−584	5.3	0.004
−100	143.6	−170.8	−495	7.0	0.007
−80	121.5	−139.1	−407	10.5	0.014
−60	99.4	−107.6	−320	19	0.037
−40	77.5	−76.4	−234	56	0.13
−20	55.8	−46.1	−152	490	0.73

specify $a<b<c$. Now the algorithm which will lead to a solution can be given in the following steps:

(1) Put lens system in position A and find the conjugates for which system will work as a zoom system.

(2) Put lens system in position C and compute error in focus.

(3) Adjust focal length of f_3 to reduce focusing error for position C and return to step 1. Repeat until focusing error is sufficiently reduced, then go to step 4.

(4) Put lens system in position B and compute error in focus.

(5) Adjust focal length of f_2 to reduce focusing error in position B and return to step 1. When focusing error is sufficiently reduced in position B, the focal lengths for f_2 and f_3 have been found.

The algorithm has been programmed as an iterative routine for a digital computer. Examples of numerical solutions are given in Tables I and II. It is interesting to note that a large magnification range is obtainable in the optically compensated case, while focal shifts are relatively modest. Clearly the optically compensated zoom lens with its single moving part is attractive from the standpoint of simplicity of construction. It should nevertheless be recognized that the linearly compensated zoom lens has certain merits tending to offset the additional mechanical complexity involved. In its most general form the elements of the linearly compensated zoom lens may all move independently, subject to the constraint that these movements are linearly related. An example where but two movements are allowed is shown in Fig. 7. Comparing Figs. 7 and 8, which have the same distance from object to image, we see that in this instance the linearly compensated example has a smaller focus error and a better distribution of power among the elements.

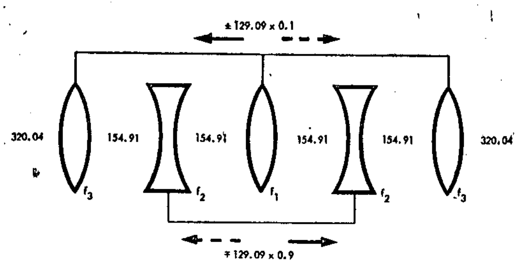

FIG. 7. Five-component system with two motions linearly related, magnification range ×8.13, maximum focusing error ≐0.0026, $f_1=289.16$, $f_2=-229.57$, $f_3=215.14$.

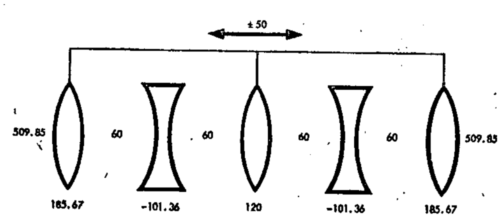

FIG. 8. Optically compensated zoom lens with same over-all length from object to image as system in Fig. 7, magnification range ×8.39, maximum focusing error = 0.0081.

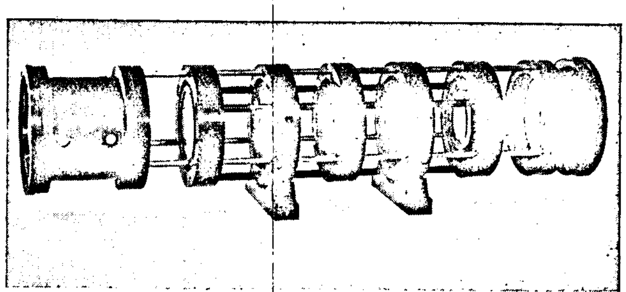

FIG. 9. Prototype of five-component optically compensated zoom lens with auxiliary end components (housing removed).

As a specific example, a five-element optically compensated zoom projection lens has been designed and fabricated and is shown in Fig. 9. The application of the lens called for a magnification range of 10, with the upper and lower limits unspecified; that is, 1–10, 3–30, etc. In this circumstance it was elected to develop a "module," ranging from $10^{-\frac{1}{2}}$ to $10^{\frac{1}{2}}$ in magnification, such that end elements could be added for later specified upper and lower limits. As has been noted, it is not possible to deviate excessively from a symmetrical construction (at midposition travel) if the maximum number of crossings is to be obtained. Acceptance then, of such a symmetrical design eliminated coma, distortion, and lateral color in the midposition.

Correction of aberrations at one extreme of travel resulted automatically in correction at the other extreme of travel. These considerations led to considerable simplification of the design task and reduced the number of tools and test plates by half, not an unimportant factor when only a few units are to be made.

The basic module covers a 112×112-mm format at unit magnification. At one end of the travel the object format is 213×213 mm, and the image format is 67×67 mm. Focal shift throughout the range is well within the Rayleigh limit. At the 3.16 power setting, axial resolution is better than 200 lines per millimeter in the object. Triplet end elements have been added, to provide a range from 3 to 30 power for projecting a $4\frac{1}{2}\times4\frac{1}{2}$ in. format on a 20×20 in. screen.

CONCLUSION

The algorithm described in this paper demonstrates a convenient means of exploring all arrangements of optically and linearly compensated zoom lenses. An upper limit to the number of crossings obtainable in terms of finite variable air spaces has been established.